Math 45 4.2 Solving Systems of Linear Equations Using Substitution

Objectives: 1) Solve a linear system with 2 equations and 2 variables using the substitution method.

Recall: The solution to a system is an ordered pair (x,y) that makes both equations true.
So we have to use both equations in our work.

Solve the system by substitution & classify.

YES ① $\begin{cases} y = -2x + 10 & \leftarrow A \\ 3x + 5y = 8 & \leftarrow B \end{cases}$

Step 1: Solve one equation for one variable.
Note: use any equation which already has an isolated variable.

A has y isolated. $y = (-2x+10)$

Step 2: Substitute for that isolated variable in the other equation.

B: $3x + 5(y) = 8$

$3x + 5(-2x+10) = 8$

Step 3: Solve the resulting equation to get a value for one variable.

$3x + 5(-2x+10) = 8$

$3x - 10x + 50 = 8$ dist

$-7x + 50 = 8$ combine

$\quad -50 \quad -50$

$\frac{-7x}{-7} = \frac{-42}{-7}$

$x = 6.$

Step 4: Substitute result from step 3 into either equation to get the other coordinate of the solution.

option 1: into A

$y = -2(6)+10$

$y = -12+10$

$y = -2$

option 2: into B

$3(6) + 5y = 8$

$18 + 5y = 8$

$-18 \quad -18$

$\frac{5y}{5} = \frac{-10}{5}$

$y = -2$

Step 5: Write solution as an ordered pair.

$\boxed{(6,-2)}$

Consistent independent

Caution: Always write x-coordinate first.

YES ② $\begin{cases} 2x - y = -15 & A \\ 4x + 3y = 5 & B \end{cases}$

step 1: When isolating a variable, you always have four options

option 1 : solve A for x → will divide by 2, creating fractions ☹

* option 2: solve A for y → no divide! ☺

option 3: solve B for x → divide by 4 ☹

option 4: solve B for y → divide by 3 ☹

$$2x - y = -15$$
$$\underline{-2x} \qquad \underline{-2x}$$
$$\frac{-y}{-1} = \frac{-2x}{-1} \frac{-15}{-1}$$
$$\underline{y = 2x + 15}$$

step 2: substitute result into other equation

B: $4x + 3(y) = 5$

$4x + 3(2x+15) = 5$

step 3: Solve.

$$4x + 6x + 45 = 5$$
$$10x + 45 = 5$$
$$\underline{-45} \quad \underline{-45}$$
$$\frac{10x}{10} = \frac{-40}{10}$$
$$x = -4$$

step 4: substitute into A, or B, or the result from step 1

result from step 1 $\quad y = 2x + 15$

$$y = 2(-4) + 15$$
$$y = -8 + 15$$
$$y = 7$$

step5: Solution $(-4, 7)$
consistent independent

maybe ③ $\begin{cases} 2x + 3y = 9 & \leftarrow A \\ x + 2y = \frac{13}{2} & \leftarrow B \end{cases}$

step 1: choose to isolate x in B. (no divide).

$$x + 2y = \frac{13}{2}$$
$$x = \left(-2y + \frac{13}{2}\right)$$

step 2: substitute into other equation.

$$2(x) + 3y = 9$$
$$2\left(-2y + \frac{13}{2}\right) + 3y = 9$$

step 3: solve

$$-4y + 2 \cdot \frac{13}{2} + 3y = 9 \qquad \text{dist}$$
$$-4y + 13 + 3y = 9 \qquad \text{simplify frac}$$
$$-y + 13 = 9 \qquad \text{combine}$$
$$\underline{-13} \quad \underline{-13}$$
$$\frac{-y}{-1} = \frac{-4}{-1}$$
$$y = 4$$

step 4: substitute into result from step 1 (or A or B).

$$x = -2(4) + \frac{13}{2}$$
$$x = -8 + \frac{13}{2}$$
$$x = \frac{-16}{2} + \frac{13}{2}$$
$$x = \frac{-3}{2}$$

step 5: $\boxed{\text{Solution } \left(\frac{-3}{2}, 4\right) \quad \text{consistent independent}}$

YES (4) $\left.\begin{aligned} x - 3y &= 5 \\ -2x + 6y &= 3 \end{aligned}\right\}$ $\begin{aligned} A \\ B \end{aligned}$

<u>step 1</u>: Isolate A for x.

$$x - 3y = 5$$
$$x = (3y + 5)$$

<u>step 2</u>: Substitute into B.

$$-2(3y+5) + 6y = 3$$

<u>step 3</u>: Solve

$$-6y - 10 + 6y = 3 \qquad \text{dist}$$
$$-10 = 3 \qquad \text{combine}$$

* VARIABLES DISAPPEARED *

This is correct work.

The result is a false statement, $-10 \neq 3$

$\boxed{\begin{array}{l}\text{no solution} \\ \text{inconsistent}\end{array}}$ ← if we graph, these are parallel lines.

YES (5) $\left.\begin{aligned} 6x - 2y &= -4 \\ -3x + y &= 2 \end{aligned}\right\}$ $\begin{aligned} A \\ B \end{aligned}$

<u>step 1</u>: Solve B for y.

$$-3x + y = 2$$
$$y = (3x + 2)$$

<u>step 2</u>: substitute into A

$$6x - 2(3x+2) = -4$$
$$6x - 6x - 4 = -4 \qquad \text{dist}$$
$$-4 = -4 \qquad \text{combine}$$

* VARIABLES DISAPPEARED *

This is correct work.

The result is a true statement, $-4 = -4$.

$\boxed{\begin{array}{l}\text{all points on the line are solutions} \\ \text{consistent dependent}\end{array}}$

YES (6) $\begin{cases} y = \frac{2}{3}x + 1 & A \\ y = -\frac{3}{2}x + 40 & B \end{cases}$

Step 1: Both A and B have y isolated.

Step 2: Substitute A into B

$A: \ y = (\frac{2}{3}x + 1)$

$B: \ (y) = -\frac{3}{2}x + 40$

$$\left(\frac{2}{3}x + 1\right) = -\frac{3}{2}x + 40$$

Note: This is effectively setting them equal.

Step 3: Solve : clear fractions by mult by LCD = 6.

$$\overset{2}{\cancel{6}} \cdot \frac{2x}{\cancel{3}} + 6 \cdot 1 = \overset{3}{\cancel{6}} \cdot \left(\frac{-3x}{\cancel{2}}\right) + 6 \cdot 40$$

$$4x + 6 = -9x + 240$$
$$+9x \qquad\qquad +9x$$

$$13x + 6 = 240$$
$$-6 \qquad -6$$

$$\frac{13x}{13} = \frac{234}{13}$$

$$x = 18$$

Step 4: Substitute into either A or B.

$A: \ y = \frac{2}{\cancel{3}}(\overset{6}{\cancel{18}}) + 1$

$y = 2 \cdot 6 + 1$

$y = 13$

Step 5: $\boxed{\begin{array}{l}(18, 13) \\ \text{consistent independent}\end{array}}$

YES 7 $\begin{cases} \frac{x}{2} + \frac{y}{3} = \frac{1}{12} & A \\ \frac{5x}{4} + \frac{2y}{3} = \frac{-11}{24} & B \end{cases}$

<u>step 0</u>: Before anything, clear fractions from both equations.

A: $\overset{6}{\cancel{12}} \cdot \frac{x}{\cancel{2}} + \overset{4}{\cancel{12}} \cdot \frac{y}{\cancel{3}} = \cancel{12} \cdot \frac{1}{\cancel{12}}$ LCD = 12, mult all terms by 12

$6x + 4y = 1$ NEW A

B: $\overset{6}{\cancel{24}} \cdot \frac{5x}{\cancel{4}} + \overset{8}{\cancel{24}} \cdot \frac{2y}{\cancel{3}} = \frac{-11}{\cancel{24}} \cdot \cancel{24}$ LCD = 24

$6 \cdot 5x + 8 \cdot 2y = -11$

$30x + 16y = -11$ NEW B.

$\begin{cases} 6x + 4y = 1 & \text{NEW A} \\ 30x + 16y = -11 & \text{NEW B} \end{cases}$

<u>step 1</u>: Looks like solving NEW A for y will have smallest denominators.

$6x + 4y = 1$

$\frac{4y}{4} = \frac{-6x}{4} + \frac{1}{4}$

$y = \frac{-3}{2}x + \frac{1}{4}$

<u>step 2+3</u>: substitute into NEW B.

$30x + 16\left(\frac{-3}{2}x + \frac{1}{4}\right) = -11$

$30x + \overset{8}{\cancel{16}} \cdot \left(\frac{-3}{\cancel{2}}\right)x + \overset{4}{\cancel{16}} \cdot \left(\frac{1}{\cancel{4}}\right) = -11$ dist

$30x - 24x + 4 = -11$

$6x + 4 = -11$ combine

$-4 \qquad -4$

$\frac{6x}{6} = \frac{-15}{6} \begin{matrix} \div 3 \\ \div 3 \end{matrix}$

$x = \frac{-5}{2}$

<u>step 4</u>: Subst into NEW A, NEW B, A, B or result from step 1.

$y = \frac{-3}{2}\left(\frac{-5}{2}\right) + \frac{1}{4} = \frac{15}{4} + \frac{1}{4} = \frac{16}{4} = 4$

<u>step 5</u> $\boxed{\left(-\frac{5}{2}, 4\right) \text{ consistent independent}}$

(8) $\begin{cases} y = \frac{3}{4}x & A \\ y = 4(x-1) & B \end{cases}$

set equal:

$\frac{3}{4}x = 4(x-1)$ dist

$\frac{3}{4}x = 4x - 4$ clear frac

$3x = 4\cdot 4x - 4\cdot 4$

$3x = 16x - 16$

$-16x \quad -16x$

$\frac{-13x}{-13} = \frac{-16}{-13}$

$x = \frac{16}{13}$

subst $y = \frac{3}{\cancel{4}}\left(\frac{\cancel{16}^{4}}{13}\right)$

$y = \frac{12}{13}$

$\boxed{\text{solution } \left(\frac{16}{13}, \frac{12}{13}\right) \quad \text{consistent independent}}$

(9) $\begin{cases} x + y = 24000 & A \\ 0.04x + 0.065y = 1360 & B \end{cases}$

clear decimals by mult by 1000

$40x + 65y = 1360000$

reduce by dividing all by 5

$8x + 13y = 272000$ NEW B.

$\begin{cases} x + y = 24000 & A \\ 8x + 13y = 272000 & \text{NEW B.} \end{cases}$

Solve A for y: $y = 24000 - x$

subst into NEW B:

$8x + 13(24000 - x) = 272000$

$8x + 312000 - 13x = 272000$

$-5x + 312000 = 272000$

$-5x = -40000$

(9) cont.

$x = \frac{-40000}{-5}$

$x = 8000$

subst into $y = 24000 - x$

$y = 24000 - 8000$

$y = 16000$

$\boxed{\text{Solution } (8000, 16000) \quad \text{consistent independent}}$

(10) $\begin{cases} \frac{2}{3}x + \frac{1}{3}y = 1 & A \\ -4x - 2y = -6 & B \end{cases}$

clear fractions in A: mult by 3.

$3\cdot\frac{2}{3}x + 3\cdot\frac{1}{3}y = 3\cdot 1$

$2x + y = 3$ NEW A

simplify B by div. by -2

$\frac{-4x}{-2} \frac{-2y}{-2} = \frac{-6}{-2}$

$2x + y = 3$ NEW B

$\left.\begin{array}{l} 2x + y = 3 \\ 2x + y = 3 \end{array}\right\}$ NEW A, NEW B

Same equation!

$\boxed{\text{all points on line are solutions. Consistent dependent}}$

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(11) $\begin{cases} 3x + 2y = 1 & A \\ x + \frac{2}{3}y = -\frac{5}{3} & B \end{cases}$

step 1: solve B for x

$$x = -\frac{2}{3}y - \frac{5}{3}$$

step 2+3: subst into A + solve

$$3\left(-\frac{2}{3}y - \frac{5}{3}\right) + 2y = 1$$

$$-2y - 5 + 2y = 1$$

$$-5 = +1$$

* VARIABLES DISAPPEAR *

false statement

$$\boxed{\begin{array}{l}\text{no solution} \\ \cdot\text{inconsistent}\end{array}}$$